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MIXING OF FINELY DISPERSED VAPORIZING PARTICLES WITH A DEFLECTING GAS STREAM

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UDC 533.6.071.08.632.57

In a number of problems of aerogas dynamics it becomes necessary to introduce finely dispersed particles into a gas stream. Such problems are connected, for example, with the design of prospective cryogenic and high-enthalpy wind tunnels [1, 2], visualizing devices [3, 4], accelerators of macroscopic particles for the investigation of surface erosion, the creation of active media for gasdynamic lasers using multiphase mixing [5, 6], etc. In this case certain demands may be placed on the distribution of parameters in the mixing zone, such as the macroparticles or the mass density of the gas formed as a result of their vaporization, the momentum of the particles, or any other aspect of their mass spectrum. In particular, the investigation of the conditions required to realize the advantages of multiphase mixing in gasdynamic lasers (GDL) [5, 6] led to the need to solve new gasdynamic problems which did not arise in the traditional devices for amplifying radiation [7, 8]. The statement and solution of a number of these problems are given in [9-11]. They include the problem of the maximum possible acceleration of aerosol particles by different gases, the problem of determining the depth of penetration of particles into a comoving stream and the time of their vaporization as a function of diameter, and problems of the influence of compression shocks and viscous effects, including detachment zones, the mutual influence of vaporization and vibrational relaxation, and some others. One of the central problems is the obtainment of the flow field in the mixing zone which is closest to uniform, since constancy of the gasdynamic parameters and concentrations of the components of the gas mixture is very important, for example, in the zone of action of a GDL resonator.

In [6, 10] the mixing problem was considered in an integral statement based on the use of conservation laws with allowance for the supply of a certain mass, momentum, and energy to the working stream. Such an approach makes it possible to determine the variation of pressure, temperature, and other parameters as a result of mixing, although the flow structure directly in the mixing zone remains indefinite. The detailed investigation of this structure is needed in connection with the finiteness of the vibrational relaxation time τ, and hence with the boundedness of the region of mixing of two streams in which population inversion and radiation amplification can occur. Equalization of the profiles of different parameters across a mixing zone must take place just in a region of $1 \lesssim u\tau$, where u is the characteristic velocity in the mixing zone. And the present

Moscow. Translated from Zhurnal Prikladnoi Mekhniki i Tekhnicheskoi Fiziki, No. 3, pp. 77-82, May-June, 1982. Original article submitted May 6, 1981.

0021-8944/82/2303-0381 $07.50

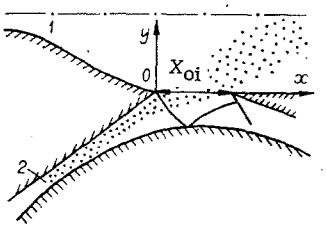

Fig. 1

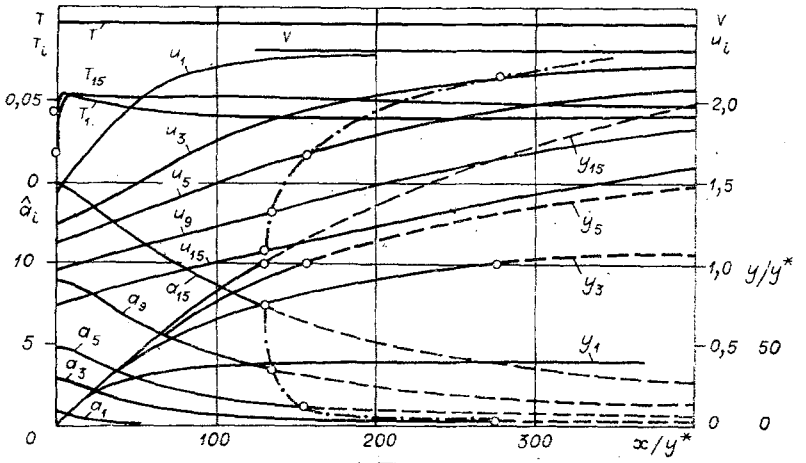

Fig. 2

report is devoted to an investigation of the structure of the mixing zone for a given particle size distribution function and to finding the conditions making the flow in this region approximate uniform flow.

Let us consider the flow scheme presented in Fig. 1. Into the supersonic working stream 1, which in the case of a GDL can consist of vibrationally excited nitrogen with a small admixture of water, aerosol particles are introduced, carbon dioxide, for example, accelerated by stream 2 to the maximum possible velocity. As shown in [11], the velocities of different fractions in the exit cross section of the acceleration nozzle differ considerably. No explicit form of the distribution function of these fractions is known from the literature for this class of problems; moreover, it varies in the process of acceleration and vaporization of the particles. In those cases when the mass of the particles comprises several percent of the total mass of the working mixture and the counter-influence of the particles on the parameters in the mixing zone can be neglected, it is expedient to set up the problem as follows. The magnitudes and directions of the velocities of particles of different diameters are assigned in the exit cross section (the O–x in Fig. 1) on the basis of data obtained on acceleration [9-11]. From these data one can assign one or another form of the size distribution function $f(\hat{a})$ or use an experimental dependence for this function. Then the particle trajectories up to total vaporization are calculated in stream 1. From the aspect of the minimization of disturbances it is advisable to consider conditions when the shock wave due to the interaction of the gaseous phases is located in stream 2 [9].

In the present work we use the model of phase transitions and the program for numerical solution of the equations of particle dynamics of [12, 13], extrapolated to the case of a multicomponent mixture under consideration. In this model the fluxes of mass, momentum, and energy to a vaporizing or condensing particle in the free-molecule regime of streamline flow are obtained under the assumptions of a locally Maxwellian velocity distribution function of gas molecules in the vicinity of the particle and of total equilibrium with respect to all degrees of freedom in their collision with the particle surface. In the continuous regime of streamline flow a model of diffusional vaporization of particles is adopted which allows for semiempirical information about the influence on this process of the blowing over the particle of the carrier gas and the influence of its compressibility. To describe the dynamics and heat and mass exchange of particles at arbitrary Knudsen numbers we adopt Sherman interpolation equations of the "parallel conductivity" type, which change asymptotically for $\hat{K}n_i \ll 1$ and $\hat{K}n_i \gg 1$ into expressions for the continuous and free-molecule regimes of streamline flow. In extrapolating this model to the case of a multicomponent medium we allow for the fact that the density of vapors of particle material is much less than the density of the passive carrier gas. We note that in the example under consideration even the largest particles in the plane-parallel stream 1 at once proved to be under conditions of free-molecule streamline flow. As an example, in Fig. 2 we present the results of a numerical investigation of the dynamics and heat and mass exchange of carbon dioxide particles with eight fractions ($\hat{a}_i$ = 1, 3, 5, ..., 15 μm) accelerated by hydrogen in the acceleration nozzle 2; the values of their parameters obtained in [11] are taken as the initial values. Here we assigned the following values of the gasdynamic quantities in stream 1: stagnation parameters p_0 = 7 MPa, T_0 = 3000°K, speed of sound in the critical cross section a_* = 1020 m/sec, γ = 7/5, $\lambda = u/a_* = 2.33$, y_* = 1 mm, molecular mass $M = 28 \cdot 10^{-3}$ kg/kmole. The above example corresponds to an injection angle $\alpha = 45°$. The trajectories $y_i(x)$ of several fractions, the variations along stream 1 (x axis) of particle sizes a_i, temperatures T_i, and longitudinal velocity components $\hat{u}_i$ are shown. We note the nonmonotonic character of the particle temperature variation: First they are heated by the

stream without vaporizing, and then they are cooled as a result of vaporization. The finest particles with $\hat{a}_i = 1\,\mu m$ are already fully vaporized not far from the injection point (in Fig. 2 the point of vaporization at $y/y_* = 100$ is marked by a light circle). Large particles can reach the opposite wall of the stream; the dashed lines are their trajectories in the case of specular reflection. The dash–dot curves connect the particle parameters a_i and u_i at the instant of striking the wall. Since the trajectories of the vaporizing particles were calculated independently for each type i, in determining the total concentration in the mixing zone it was assumed that different types are not influenced by each other's vaporization. To avoid difficulties connected with the transition of particles from one fraction to another during vaporization, the summation was also carried out independently over all types i, assigned preliminarily with one or another weight in the initial cross section. The condition of the absence of a counterinfluence of particles on the gas parameters was also important for the application of the "superposition principle" in the indicated sense.

The results of the numerical investigation indicate that the particle radius $\hat{a}(\hat{a}_0, y)$ is a complicated function of the initial value and the coordinate.

First let us consider the approximate statement and solution of the inverse problem, according to which the particle size distribution function is chosen from the requirement of uniformity of the vapor concentration in a certain cross section of the working zone. Let particles with a radius a_{0i} be introduced into the stream in the plane $y = 0$ along the section $X_{0i}(a_{0i})$ with a normal velocity $v_{0i}(a_{0i})$ (see Fig. 1). Then, neglecting fragmentation and coagulation, we can write the equation of conservation of the number of particles a_i of each type in the form

$$v_{0i} n_{0i} X_{0i} = v_i(y) n_i(y) X_i(y),$$

where n_i is the number of particles of type i per unit volume.

Then the density distribution $\rho(y)$ in a plane $y = \text{const}$ of the vapor formed as a result of the vaporization of all fractions has the form

$$\rho(y)\, u = -\sum_i \frac{\partial m_i}{\partial y} n_i(y)\, v_i(y)\, X_i(y) = -\sum_i \frac{\partial m_i}{\partial y} n_{0i} v_{0i} X_{0i}.$$

Changing from a discrete distribution to a continuous function $f(\hat{a}) = d\hat{n}/d\hat{a}$, we obtain

$$\rho(y) u = -\int_{a_0^*(y)}^{\hat{a}_{0\max}} X_0(\hat{a}) \frac{\partial \hat{m}(\hat{a}_0, y)}{\partial y} \hat{v}_0(\hat{a}_0) f_0(\hat{a}_0)\, d\hat{a}_0, \tag{1}$$

where the integral is taken over the entire set of particles which were unable to vaporize up to the plane y. This information is obtained preliminarily from a calculation of the trajectories of the vaporizing particles. To obtain the analytical form of the function $f_0(\hat{a}_0)$ in the insertion cross section we assume for simplicity that all particles are accelerated to the same initial velocity $v_{0i} = \hat{v}_0$ and the angles of their insertion into the working stream are the same. We designate $X_{0i} = L$, and then Eq. (1) takes the form

$$\rho(y) = -\frac{L}{u} \hat{v}_0 \int_{a^*(y)}^{\hat{a}_{0\max}} \frac{\partial \hat{m}(\hat{a}_0, y)}{\partial y} f_0(\hat{a}_0)\, d\hat{a}_0. \tag{2}$$

This is a homogeneous integral equation of the Volterra type for the unknown function $f_0(\hat{a}_0)$. Its solution depends on the entire collection of physical processes taking place with the aerosol particles as they move in stream 1. The function $\hat{m}(\hat{a}_0, y)$ of interest to us for the example presented in Fig. 2 is shown in Fig. 3. It is seen that in the initial section of insertion into the supersonic stream 1 the intensity of vaporization of all particles is negligibly small owing to their low initial temperature ($T_{0i} \approx 100°K$). Then in the course of heating (upper part of Fig. 2) the pressure of saturating vapor above the particles grows sharply and the vaporization process accelerates. It follows from Fig. 3 that the functions $a_i(y)$ can be approximated by different functions. A parabolic function which allows for the initial section of particle heating almost without vaporization is qualitatively suitable. For example, let the particles vaporize in accordance with the law

$$\hat{a} = \hat{a}_0 - \hat{a}_0^*(y), \quad \hat{a}_0^*(y) = \xi y^2.$$

Then $\partial \hat{m}/\partial y = -8\pi\rho^0(a_0 - \xi y^2)^2 \xi y$, where ρ^0 is the density of the particle material. Further, we assume that the maximum radius $\hat{a}_{i\max}$ goes to infinity, while particles which have struck the wall are out of consideration. Then Eq. (2) has the form

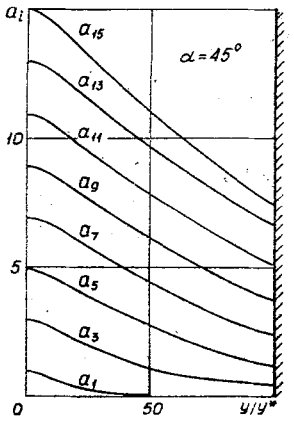

Fig. 3

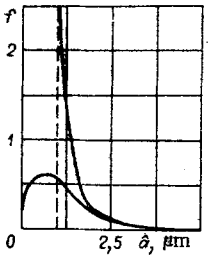

Fig. 4

$$\rho(y) = \frac{L}{u}\hat{v}_0 8\pi\rho^0\xi\int\limits_{\xi y^2}^{\infty}(\hat{a}_0 - \xi y^2)^2\, y f_0(\hat{a}_0)\, d\hat{a}_0.$$

We shall seek its solution in the form $f(\hat{a}_0) = \beta\hat{a}_0^n$. Performing the integration, we obtain the expression

$$\rho(y) = -\beta y\left(\frac{\xi^{3+n}y^{6+2n}}{3+n} - 2\xi y^2\frac{\xi^{2+n}y^{4+2n}}{2+n} + \xi^2 y^4\frac{\xi^{1+n}y^{2+2n}}{1+n}\right),$$

from which it follows that for the condition of independence of the vapor density on y we must set $n = -7/2$. In this case

$$\rho = \rho_c = \frac{16}{15}\frac{L}{u}\hat{v}_0 8\pi\rho^0\sqrt{\xi}\beta. \tag{3}$$

where ρ_c is a quantity which is known from the initial conditions.

Having determined the scale factor β from Eq. (3), we obtain the final expression for the distribution function

$$f_0(\hat{a}_0) = \frac{15\rho_c u}{2^7\pi L\hat{v}_0\rho^0\sqrt{\xi}\,\hat{a}_0^{7/2}}. \tag{4}$$

It follows formally from the means of obtaining Eq. (4) that uniformity along y is reached asymptotically, i.e., $x \to \infty$. The values of x which are closest to the point of particle insertion satisfying the condition $x < u\tau$ for the class of problems under consideration, where τ is the time of vibrational relaxation of the working gas mixture, are of practical interest. Nevertheless, the result obtained is interesting in that it shows the fundamental possibility of achieving uniformity in a vaporizing disperse medium without the mechanism of molecular diffusion. It is clear that if the particle size distribution function differs strongly from (4), then in the absence of diffusion there willl be no uniformity along y, generally speaking, even as $x \to \infty$. If we allow for the results of [11] on the insignificant variation in particle size during motion in the acceleration nozzle, then the mass spectrum which must be obtained before acceleration will differ from the distribution (4) just found only by a scale factor (constant for all particles).

The function (4) grows sharply in the direction of small particles. The distribution function f_1 of aluminum oxide particles [14] which was used earlier [9] in an analysis of the acceleration problem is shown in Fig. 4 for comparison. It differs sharply from the function (4) in the region of small sizes, but one must not forget that the causes of particle formation are entirely different in the case of [14]. As shown in [11], with a decrease in the initial size the vaporization intensity in the acceleration nozzle grows, which can result in degradation of the carrier properties of the light gas. Therefore, it is desirable to limit the particle spectrum on the side of small values to a certain least radius $\hat{a}_{0\min}$ (possible limits on $\hat{a}_{0\max}$ are discussed in [9]). Such a limit results, in turn, in the fact that a thin layer near the wall in the vicinity of insertion into the mixing zone will not be "seeded" if the effect of fragmentation of larger particles in the region of the insertion section is insignificant. On the whole, the absence of fine particles is favorable, since dissipative phenomena exist in the layer near the wall (boundary layer, detachment zones, etc.), and this region is excluded from the main working core anyway.

We note that the above procedure also contains the solution of the direct problem, since the concentration distribution of the vaporizing component in different cross sections of the working zone can be determined on the basis of calculations analogous to those presented in Figs. 2 and 3, in which each particle type i can be taken into account with a different weight, which corresponds to different size distribution functions assigned in the initial cross section O–x (see Fig. 1).

Thus, the results of the present work and [9-13] allow us to formulate the variational problem of determining the set of conditions (the initial mass spectrum of the dispersing particles and the parameters of the gas in the subsonic part of the acceleration nozzle, the angle of encounter with the main stream, etc.) providing an extremum of a certain functional of the parameters of the physical situation occurring in the mixing of high-velocity two-phase streams.

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